

Kinetic Gravity Braiding and axion inflation

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We constructed a new class of inflationary model with the higher derivative axion field which obeys constant shift symmetry. In the usual axion (natural) inflation, the axion decay constant is predicted to be in the super-Planckian regime which is believed to be incompatible with an effective field theory framework. With a novel mechanism originating from a higher derivative kinetic gravity braiding (KGB) of an axion field we found that there exist a huge parameter regime in our model where axion decay constant could be naturally sub-Planckian. Thanks to the KGB which effectively reduces the Planck constant. This effectively reduced Planck scale provides us the mechanism of further lowering down the speed of an axion field rolling down its potential without introducing super-Planckian axion decay constant. We also find that with that wide range of parameter values, our model induces almost scale invariant power spectrum as observed in CMB experiments.

Inflation is an exponential expansion phase of our universe in its very early stage of evolution. Even though this is by far the only successful mechanism to solve several problems in stand Big-Bang model of the universe, we still do not have a fundamental theory which leads to such a mechanism. In order to realise such an exponential expansion, often a scalar field is invoked with an unnaturally flat potential which has already been proved to be very difficult to construct in the quantum field theory framework. However It has been well accepted that shift symmetry plays a very crucial role in the inflationary dynamics. It is this symmetry which keeps the potential sufficiently flat to realise inflation. In this respect usual standard model of particle physics could still be a natural framework to study inflation. A pseudo scalar field called axion may play an important role in this regard. This is a hypothetical field associated with the Peccei-Quinn symmetry which has been introduced to solve the strong CP problem in QCD in standard model of particle physics. This axion field obeys shift symmetry. By using this axionic shift symmetry a "natural" inflation had been proposed in [1]. In spite of its viability, observation suggests that axion decay constant should be $f \geq 3M_p$. Question has been raised on this large f in the effective field theory framework [2] and also quantum gravity effect may also ruin the axion symmetry at that scale [3]. Subsequently various generalization of the above natural inflation scenario has been made [4]. Very recently some viable phenomenological extension of this natural inflation with the sub-Planckian axion decay constant have also been proposed [5] which leads to the resurgence of interest in this subject. In this letter we will construct another viable model of axion inflation which is relying on the higher derivative kinetic gravity braiding. There has been lot of studies based on this kind of model in the

context of inflation which goes by the name of G-inflation [6] and also in the context of dark energy mode building [7, 8]. We will very closely follow those construction in this letter. The essential mechanism which has already been pointed out in [6] that the kinetic braiding parameter is playing the role of flattening the potential in certain region of parameter space. We will see that in that range of parameter space we can make our axion decay constant f to be sub-Planckian by appropriately choosing another sub-Planckian scale s associated with the kinetic gravity braiding (KGB) of our model.

We start with the following action

$$\mathcal{L} = \frac{M_p^2}{2}R - X - M(\phi)X\Box\phi - \Lambda^4 \left(1 - \cos\left(\frac{\phi}{f}\right)\right) \quad (1)$$

where $X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ and $\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu)$. f is the axion decay constant. Λ is related to the axionic shift symmetry breaking scale. We call the term associated with the higher derivative action as KGB following [7]. One of the interesting properties of this higher derivative term is that it does not lead to any unwanted degrees of freedom.

Assuming the usual FRW Metric ansatz

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \quad (2)$$

we obtain the following Einstein equation for the scale factor a

$$H^2 = -H\dot{\phi}^3 M(\phi) - \frac{X}{3} + \frac{2}{3}X^2 M'(\phi) + \frac{\Lambda^4}{3} \left(1 - \cos\left(\frac{\phi}{f}\right)\right) \quad (3)$$

and for the axion field

$$\begin{aligned} \frac{1}{a^3} \frac{d}{dt} \left[a^3 \left(1 - 3HM\dot{\phi} - 2M'X\right) \dot{\phi} \right] &= \partial^\mu\phi\partial_\mu(M'X) \\ &- \frac{\Lambda^4}{f} \sin\left(\frac{\phi}{f}\right) \end{aligned} \quad (4)$$

Where, $H = \dot{a}/a$ is the Hubble constant.

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Following the reference [6] if we consider slow roll condition, the scalar field equation turns out to be

$$3H\dot{\phi}\left(1-3M(\phi)H\dot{\phi}\right)+\frac{\Lambda^4}{f}\sin\left(\frac{\phi}{f}\right)=0 \quad (5)$$

We assume that the inflation is driven by the KGB such that the function $M(\phi)$ satisfies $|M(\phi)H\dot{\phi}| \gg 1$. This condition will lead us to

$$\tau = \frac{M(\phi)\Lambda^4}{f}\sin\left(\frac{\phi}{f}\right) \gg 1. \quad (6)$$

Once above condition is satisfied, the expressions for slow roll parameters turn out to be

$$\epsilon = \frac{M_p^2}{2f^2\sqrt{\tau}} \frac{\sin\left(\frac{\phi}{f}\right)^2}{\left(1-\cos\left(\frac{\phi}{f}\right)\right)^2}; \quad \eta = \frac{M_p^2}{2f^2\sqrt{\tau}} \frac{\cos\left(\frac{\phi}{f}\right)}{\left(1-\cos\left(\frac{\phi}{f}\right)\right)}$$

$$\alpha = \frac{M_p}{36} \frac{M'}{M} \left(\frac{4\epsilon^2}{\tau}\right)^{\frac{1}{4}}; \quad \beta = M_p^2 \frac{M''}{M} \left(\frac{4\epsilon^2}{\tau}\right)^{\frac{1}{2}} \quad (7)$$

As one can see from the above expressions for the slow roll parameters that KGB function M flattens the axion potential in term of τ . As we will see this particular novel effect of KGB will help us to lower the axion decay constant f into the sub-Planckian regime. The condition eq.(7) with $\sin\left(\frac{\phi}{f}\right)$ function also tells us that in order to maintain those slow roll condition inflation driven by KGB has to happen not very close to the maximum of the potential but little away from the maximum such that $\sin\left(\frac{\phi}{f}\right) \simeq \mathcal{O}(1)$. We will see in our subsequent analysis that this is indeed the case.

Keeping in mind the periodic nature of the potential we will study following two different choices of braiding functions

Model-I: For $M(\phi) = \frac{1}{s^3}$, where for our subsequent discussion we fix $s > 0$ and calling it as our new KGB scale, we get

$$\tau_I = \tau_0 \sin\left(\frac{\phi}{f}\right) \gg 1; \quad \alpha_I = 0; \quad \beta_I = 0, \quad (8)$$

where we define $\tau_0 = \Lambda^4/(s^3 f)$. This also says that with this particular choice, inflation driven by KGB happens in region I of the potential as shown in the above Fig.1. As one can see in this region of the potential speed of axion field $\dot{\phi} < 0$. This could further be checked by doing perturbation analysis [6] that the solution in this region is also stable with the stability condition $M\dot{\phi} \simeq \dot{\phi} < 0$.

Model-II: On the other hand if we consider $M = \frac{1}{s^3} \sin\left(\frac{\phi}{f}\right)$, we get

$$\tau_{II} = \tau_0 \sin\left(\frac{\phi}{f}\right)^2 \gg 1; \quad \alpha_{II} = -\frac{M_p}{f} \cot\left(\frac{\phi}{f}\right) \left(\frac{4\epsilon^2}{\tau}\right)^{\frac{1}{4}}$$

$$\beta_{II} = \frac{M_p^2}{36f^2} \left(\frac{4\epsilon^2}{\tau}\right)^{\frac{1}{2}} \quad (9)$$

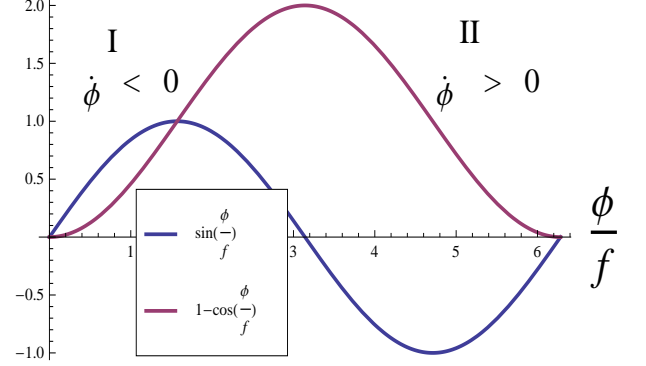


FIG. 1: The potential $V(\phi)$ and $V'(\phi)$ up to a constant factor related to the amplitude. For the **Model-I**, inflation due to KGB occurs in the region I. For **Model-II**, inflation due to KGB happens on both side of the potential namely region (I, II). One can also check that for both the models stability condition $M(\phi)\dot{\phi} < 0$ is satisfied. We assume axion always rolling down the potential

In this case, on both region (I,II) of the potential (see fig.1) inflation driven by KGB occurs. In this model also in both regions stability condition $M(\phi)\dot{\phi} \simeq \sin\left(\frac{\phi}{f}\right)\dot{\phi} < 0$ is satisfied.

Similarly, one can choose various possible periodic function for $M(\phi)$ which may have different interesting phase structure of the inflationary dynamics on various part of the axion potential. We will do our detailed study on those various choices and their dynamics in our future publication.

This is also interesting to note that for both the models we have introduced, near the maximum of the potential τ is very small which is essentially referring (see eq.(5,9)) to the usual slow roll inflationary phase of the universe rather than inflation driven by KGB. Here we would like to emphasize that if we set our model parameter values such that inflationary phase due to KGB satisfies the CMB observation, then depending on the onset value of the axion potential, one can have different types of inflationary phase along the axion potential with different effect on the perturbation. One interesting dynamics would be if onset of inflation happens near the maximum of the potential then usual slow roll inflation happens followed by the inflation driven by KGB. Due to different types of inflationary phase along the axion potential it can potentially influence the dynamics of various modes of the cosmological perturbation in a different manner. This can help us to shed some light on the problem of CMB anomaly at large angular scales which are associated with the long wavelength modes of inflationary perturbation. We will defer this study in detail in our future publication.

If we concentrate on the region of the potential where

$\sin\left(\frac{\phi}{f}\right)$ being very close to unity, then for both the model under consideration, the condition of KGB driven axion inflation turns out to be

$$\tau_I \approx \tau_{II} \approx \tau_0 = \frac{\Lambda^4}{s^3 f} \gg 1 \implies s \ll \left(\frac{\Lambda^4}{f}\right)^{\frac{1}{3}} \quad (10)$$

So we have enough region of the parameter space where the axion decay constant could be sub-Planckian along with the KGB scale. In the subsequent analysis we will derive this scale dependence more rigorously taking into account the dynamics of cosmological perturbation.

The region of parameter space could be constrained by the dynamics of fluctuation of the axion field which is directly related to the CMB power spectrum $P_{\mathcal{R}}$ associated with curvature perturbation \mathcal{R} and spectral index n_s . The expression for those quantities can be straightforwardly calculated as [6]

$$P_{\mathcal{R}} = \frac{3\sqrt{6}}{64\pi^2} \frac{H^2}{M_p^2 \epsilon} \quad ; \quad n_s = 1 - 6\epsilon + 3\eta + \frac{\alpha}{2} \quad (11)$$

Now let us study the slow roll parameters with respect to both models that we have discussed above. For both the models the explicit form of the spectral index turn out be

$$\begin{aligned} n_s^I &\simeq 1 - \frac{3}{\mathcal{A}} \frac{\sin\left(\frac{\phi}{f}\right)^{\frac{3}{2}}}{\left(1 - \cos\left(\frac{\phi}{f}\right)\right)^2} + \frac{3}{2\mathcal{A}} \frac{\sqrt{\cos\left(\frac{\phi}{f}\right) \cot\left(\frac{\phi}{f}\right)}}{\left(1 - \cos\left(\frac{\phi}{f}\right)\right)} \\ n_s^{II} &\simeq 1 - \frac{3}{\mathcal{A}} \frac{\sin\left(\frac{\phi}{f}\right)}{\left(1 - \cos\left(\frac{\phi}{f}\right)\right)^2} + \frac{3}{2\mathcal{A}} \frac{\cot\left(\frac{\phi}{f}\right)}{\left(1 - \cos\left(\frac{\phi}{f}\right)\right)} \\ &\quad - \frac{1}{2\sqrt{\mathcal{A}}} \cot\left(\frac{\phi}{f}\right) \sqrt{2\epsilon_{II}}. \end{aligned}$$

For our future convenience we have further defined a new constant $\mathcal{A} = \sqrt{\tau_0}(f/M_p)^2$ in the above expressions. One can clearly see from the expression of \mathcal{A} that usual spectral tilt $n_s - 1$ of axion (natural) inflation is reduced by a factor of $\sqrt{\tau_0}$ or in other words it essentially suppress the Planck scale. We will see that this effectively reduced Planck scale is playing the main roll in bringing down the axion decay constant f to be in the sub-Planckian regime.

In order to solve the homogeneity and flatness problem of the usual Big-Bang model, we need to have sufficient amount of inflation. This sufficient inflation is measured by so called e-folding number $\mathcal{N} = \int_{t_1}^{t_2} H dt$. From the current cosmological observations the constraint on $\mathcal{N} \approx 60$. So further constrain on our model parameters will come from this e-folding number. The analytic expressions for \mathcal{N} for both the models under consideration

\mathcal{A}	$x_1 = \frac{\phi_1}{f}$	$x_2 = \frac{\phi_2}{f}$	n_s^I	$x_1 = \frac{\phi_1^{II}}{f}$	$x_2 = \frac{\phi_2^{II}}{f}$	n_s^{II}
245	1.10084	0.298338	0.970933	1.17613	0.365871	0.98474
215	1.15973	0.314236	0.970878	1.23119	0.382151	0.98299
185	1.23134	0.333563	0.970796	1.29809	0.401779	0.98086
125	1.43922	0.389657	0.970436	1.49279	0.457857	0.97474
65	1.86299	0.504365	0.968657	1.89973	0.569322	0.96325
35	2.36077	0.642477	0.961481	2.42603	0.69966	0.94990

TABLE I: For both the model under consideration, some specific values of the parameter \mathcal{A} which provides us successful inflation driven by KGB and their possible values of the spectral index.

turn out to be,

$$\begin{aligned} \mathcal{N}_I &= -\mathcal{A} \left(2\sqrt{\sin x} + \frac{\sqrt{2}\sqrt{1+\sin x}}{\cos \frac{x}{2} + \sin \frac{x}{2}} \right. \\ &\quad \times \text{EllipticF} \left[\sin^{-1} \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right), \frac{1}{2} \right] \Big|_{x_1}^{x_2} \\ \mathcal{N}_{II} &= \mathcal{A} (x - \sin x) \Big|_{x_1}^{x_2}, \end{aligned} \quad (12)$$

where we define $x = \phi/f$. In the above expressions, the upper limit on the axion field $x_2 = \phi_2/f$ will come from the slow roll parameter. As one can imagine that inflation ends when the slow roll parameter $\epsilon = 1$ which provides us the upper limit. Furthermore if we set $\mathcal{N} \approx 60$, we can constrain the parameter space of $(\mathcal{A}, \frac{\phi_1}{f})$. This in turn will constrain the value of the spectral index. In the above Table-I we provide some possible numerical values of \mathcal{A} for which we found the values of $(\phi_1/f, \phi_2/f, n_s)$ for both the models. As one can see that the values so obtained for the spectral index n_s are close to the observed value from WMAP.

Now according to WMAP observations, considering the expression for **Model-I**, we know

$$P_{\mathcal{R}}^I = \frac{\mathcal{A}\sqrt{6}}{32\pi^2} \left(\frac{\Lambda}{M}\right)^4 \frac{(1 - \cos x_1)^3}{\sin x_1^{\frac{3}{2}}} \simeq 2.4 \times 10^{-9}. \quad (13)$$

For a fixed value of \mathcal{A} the above equation (13) can further provide us a constrain on the value of the axionic symmetry breaking scale Λ . As for example if we consider $\mathcal{A} = 65$, from the above expression eq.13 we found $\Lambda_I = 5.08 \times 10^{-3}$ in Planck unit for **Model-I**. With this value of Λ_I one can choose one set of values for $\{f_I, s_I\} = \{10^{-2}, 1.634 \times 10^{-5}\}$ in Planck unit such that all the above bounds are satisfied with the cosmological observations. A similar estimate can be done for **Model-II** where we found $\{\Lambda_{II}, f_{II}, s_{II}\} = \{4.99 \times 10^{-3}, 10^{-2}, 1.136 \times 10^{-5}\}$ in Planck unit. So one can clearly see that axion decay constant f as well as the KGB scaling parameter s simultaneously could be several order of magnitude lower than the Planck mass in order to met observational constraints. In addition another interesting outcome of our construction is that we

are getting sufficient amount of inflation with the value of axion field well below the Planck mass as well. For example with the above choices of parameters we have $\phi_1^I \approx \phi_1^{II} \approx 1.9 \times 10^{-2}$ in Planck unit. Above estimation depends on particular choice of parameters $\{f, s\}$. In principle one has large number of choices for $\{f, s\}$ as

$$\frac{s^3}{f} = \frac{1}{\mathcal{A}^2} \frac{M_p^2}{\Lambda^4} \quad (14)$$

In order to totally fix our model parameters we need to have one more observable quantity which has non-trivial dependence on the axion decay constant f . For this non-gaussianity would be one of the interesting observables which we defer for our future study. However with the above derived constraint in what follows we will discuss about another cosmological observable quantity related to tensor perturbation. As one can see from the equation below, once we fix the value of \mathcal{A} and scalar spectral index n_s , it also fixes the tensor spectral index n_T as well as tensor-to-scalar ratio as follows [6]

$$n_T = -\frac{1}{\mathcal{A}} \frac{\sin\left(\frac{\phi}{f}\right)^{\frac{3}{2}}}{\left(1 - \cos\left(\frac{\phi}{f}\right)\right)^2}$$

$$r = -\frac{32\sqrt{6}}{9} \{n_T^I, n_T^{II}\} = \{0.0757, 0.0773\} \quad (15)$$

So, the value of tensor-to-scalar ration r for both the models are very small to be detectable in near future. We have also checked that as we increase the value of \mathcal{A} , r also increases. Important point to note that perturbation in the tensor sector does not provide us further constraint on our model parameters. We, therefore, are left with one free parameter which can probably be fixed if we go beyond the linear cosmological perturbation theory.

In this letter we have discussed a new model of axion inflation which includes a specific form of higher derivative terms in consistent with the shift symmetry. Our model is strongly motivated by the recent studies on galileon scalar field theory first introduced in [9]. The specific form of the higher derivative term called kinetic gravity braiding is playing the crucial role in our model. Interesting point to note that this particular form of higher derivative term has a property that it does not introduce any ghost which generally appears in a higher derivative theory. We have seen that this particular form of higher derivative term helps us to construct a successful axion inflation model with sub-Planckian axion decay constant. One of the main problem in a standard axion (natural) inflation model is that the axion decay constant f turned out to be above the Planck scale in order met CMB observations. Through out our current analysis we have shown that this problems can be easily circumvented by introducing a higher derivative so called KGB term in the action for an axion field. This particular KGB term is playing the role in pushing the axion decay scale f

into the sub-Planckian regime. The physical reason behind this mechanism is coming from the fact that KGB parameter effectively reduces the Planck constant which in turn makes the speed of the rolling axion field along its potential slower. According to our model we also find a huge parameter region where inflation driven by KGB occurs with almost scale invariant power spectrum which has already been observed in the WMAP experiment.

We also would like to stress upon the fact that in the linear regime of cosmological perturbation theory, we could not constrain all our model parameters. What we infer from our analysis is that once we fix the value of our combined parameter \mathcal{A} from the observed spectral index $n_s \geq 0.964$ and axion shift symmetry breaking scale Λ from the observed amount of primordial fluctuation $\sim \sqrt{P_R} \approx 10^{-5}$, we can fix only the ratio s^3/f from eq.14 of two scales. So we can clearly see a huge range of values for $\{f, s\}$ where both are sub-Planckian. Interestingly enough we obtain sufficient inflation with the axion field value well below the Planck constant. These are our main result in this letter. In order to further constrain the parameters we need to go beyond the linear regime like non-gaussianity would be one of such effects. We defer this analysis for our future study.

Note added: We overlooked the reference [10] in which similar model has been considered but their conclusion is different than ours.

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